PROPOSALS FOR UPDATING TAI ALGORITHM

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Abstract

The conditions of computation of TAI and UTC are in constant evolution. The replacement of clocks of older design by new ones of type HP 5071A, started in 1993, continues with consequent improvement in the stability of the free atomic time scale EAL, the first step in the calculation of TAI. To further improve the stability of EAL and to further reduce the delay of access to TAI and UTC, the algorithm which produces them may need to be revised. With this in view, experiments on real clock data collected at the BIPM have been carried out to show the advantage of simultaneously using an upper limit on relative weights, rather then one on absolute weights, and a basic interval of computation of one month, rather than one of two months. Results of these tests are positive, so the BIPM reported on these studies to the Working Group on TAI of the Comité Consultatif du Temps et des Fréquences (CCTF) in view of implementing consequent changes in January 1998. A decision is being made.*

INTRODUCTION

Since the end of 1992, the quality of the timing data received at the BIPM has evolved rapidly thanks to the wide use of GPS time transfer and to the extensive replacement of older designs of commercial clocks by the new HP 5071A clocks. Consequently, the stability of the free atomic time scale EAL, the first step in the calculation of TAI, has improved significantly. The medium-term stability of EAL, expressed in terms of the Allan standard deviation σ_y, is estimated to be 1.3×10^{-15} for averaging times of about 40 d. This improves the predictability of UTC for averaging times of between 1 and 2 months, a scale attribute of fundamental importance for institutions charged with the dissemination of real-time time scales.

For further improvement, the stability algorithm which produces EAL may need to be revised. Given this prospect, several changes, all guided by physical considerations, have been brought to the existing algorithm. These make it possible to compute, in parallel with the published EAL, an experimental time scale E, using the real clock data collected at the BIPM from January 1996 until August 1997.

* *At its 1997 meeting, the Comité International des Poids et Mesures (CIPM) decided to change the name of the Comité Consultatif pour la Définition de la Seconde (CCDS) to that of Comité Consultatif du Temps et des Fréquences (CCTF).*

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The experimental algorithm uses the same defining equations and same fundamental procedure as the usual algorithm [1, 2, 3, 4], but the definitive computation time of the time scale is shortened from two months to one month. Another change is the imposition of an upper limit on the relative weights attributed to contributing clocks, rather than one of absolute weights.

In the following, we recall some fundamental features of the usual algorithm, then we explain the advantages of implementing the above changes and outline some possible objections. The results of the experiment carried out at the BIPM since January 1996 are then analyzed in terms of stability and accuracy.

USUAL ALGORITHM

The free atomic time scale EAL is basically a weighted averaged of data from comparison of clocks, mostly of commercial type, maintained in national timing laboratories [1]. Though timing measurements are taken and reported to the BIPM for MJDs ending in 4 and 9, *i.e.* for dates separated by 5 d, the computation treats as a whole two-month blocks of data. The reference time scale TAI is thus a deferred-time time scale available to user with a delay which can reach two months.

Treating two-month blocks of data as a whole means that the weight p_i of a given clock H_i is maintained constant over a given two-month interval and cannot change before the next one. For a given interval of computation the weight p_i, referred to as 'absolute weight', is written as:

$$p_i = \frac{C}{\sigma_i^2},$$

where σ_i^2 is the variance (over one year) of the mean frequencies of clock H_i, relative to the time scale, and estimated over two-month periods. C is a constant, the value of which is arbitrary since the contribution of clock H_i to the weighted average, referred to as 'relative weight', is given by:

$$\omega_i = \frac{p_i}{\sum_i p_i}$$

and thus is independent of the value of C.

The weighting procedure introduces an upper limit of absolute weights, p_{MAX}, such as, if over a given two-month interval

$$p_i \geq p_{MAX}, \text{ then } p_i = p_{MAX}.$$

A given clock thus receives the maximal weight according to its own stability independently of the stability of the other clocks constituting the ensemble. Keeping the value p_{MAX} constant makes the maximum contribution ω_{MAX} of a given clock vary with time.

WHY CHANGE THE ALGORITHM?

The reason for changing the algorithm is very natural: the quality of the contributing clocks has changed and the algorithm should be adapted.

The most important change is the massive replacement of older designs of clocks by the new HP 5071A clocks which are more stable and which reach their maximum of stability for shorter averaging time. Fig. 1 shows the typical stability curve of these clocks with a flicker floor level of about 6×10^{-15} for averaging times in between 20 and 40 days [5].

Since January 1993, we have observed the progressive entry of about 100 HP 5071A units in the computation and nearly all of them have been assigned the upper weight. No discrimination among these stable clocks is thus effected with the consequence that full advantage of the best ones is not taken. We alerted the CCDS Working Group on TAI to this problem, and the decision was taken in March 1995 [6] to rise the upper limit of absolute weight by a factor 2.5 on 2 May 1995. This induced a punctual increase of the maximum contribution ω_{MAX} on this date (see Fig. 2) above a general trend which is a decrease from 1.4% to 0.7%. The problem is now reappearing and we suggest that a limit should be imposed on the relative contribution made by any clock to the scale.

Another point concerns the two-month period of observation for frequency estimation. This does not appear to be optimum with a set of contributing clocks which globally present their best stability for smaller averaging times. We thus suggest to shorten the computation time of the scale to one month.

UPPER LIMIT OF RELATIVE WEIGHTS

The detailed computational process used to implement an upper limit of relative weights in the algorithm is given in Ref. 4.

ADVANTAGES

- The most important feature of this process is that it does not independently assign a weight to each clock, rather the set of clocks is treated globally. With the progressive entrance of very stable clocks fixing an upper limit of relative weight removes from the highest weight category some of those with the weakest stability. The time scale thus relies more heavily upon the very best clocks.
- This new weight determination is also more robust. One can reasonably expect that the stability of clocks contributing to TAI will still improve in coming years. This will be correctly handled by an algorithm which selects the best clocks, the criteria of stability for reaching the upper relative weight becoming more severe.
- The use of an upper limit of relative weights is already implemented in some algorithms for the generation of local time scales [3] and this technique has already proved its efficiency. In addition, following the suggestion of using an upper relative contribution for individual clocks expressed at the meeting of the CCDS Working Group on TAI in 1995, several experimental studies have already been carried out on real clock data collected at the BIPM over different periods, and have produced successful results [4].

ONE CONSTRAINT

At present, the number of clocks receiving the upper limit of absolute weight is increasing from one computation to the next with a consequent decrease of the upper contribution. At the date of implementation of the upper limit of relative weight, we will have to fix this limit in continuity with the previous computation. If we wait too long, this value will be too small to discriminate efficiently among the best clocks. For this reason we propose the implementation of the alternative algorithm as soon as possible (January 1998).

SHORTENING OF THE COMPUTATION TIME

The change proposed here is to shorten the computation time for TAI from two months to one month [7] keeping this computation time in phase with the calendar months (the interval of computation then has a length of 30 d or 35 d).

ADVANTAGES

- This shortening corresponds to an adaptation of the algorithm to the statistical properties of the clocks we have at our disposal.
- The delay of access to the time scale is reduced. The definitive computation for any month is available by the 13th of the following month. The procedure used until now (provisional results one month in two) is abandoned and each issue of *Circular T* provides final values. The question of reducing the delay of access is important since it facilitates procedures for UTC prediction implemented in time laboratories charged with the dissemination of real-time time scales.
- The work of the BIPM Time Section is simplified, a non-negligible point if the decision to update TAI every 2 or 3 d, rather then 5 d, is taken in future.

ONE PRACTICAL CONSTRAINT

Time transfer and clock data used in the TAI computation covering month n must necessarily reach the BIPM at the beginning of month $(n+1)$, since the final computation is made some days later. Data arriving too late cannot be included in the computation and those working in time laboratories should be aware of it.

ONE OBJECTION

Shortening from two months to one month the averaging time for estimation of clock frequencies, and of their variances, optimizes the stability of the time scale over a duration of one month, which may appear to conflict with the requirement of long-term stability for reference time scales. This argument has been developed fully in the past, but the current approach to the problem is different:

- The requirement is high predictability for the reference time scale in order to implement an efficient steering of local representations of UTC. Timing laboratories thus need high stability between successive computations and rapid access.
- The long-term stability of the scale mainly relies upon its accuracy, *i.e.* the quality of the measurements provided by primary frequency standards. This is on a promising path: several very accurate primary frequency standards are under development with expected type B standard uncertainties of several parts in 10^{15}, and frequency transfer techniques, such as those using two-way or GPS phase measurements, are expected soon to reach the level of 1×10^{-15} over one day.

THE EXPERIMENT

The two changes detailed above have been implemented in our usual algorithm for computation of an alternative free atomic time scale E, using real clock data collected at the BIPM from January 1996 to August 1997.

The starting date of computation (MJD = 50079, 28 December 1995) corresponds to the first date of implementation of the 5 d, rather than 10 d, recurrence of TAI updates. Each month, clock frequencies are estimated as slopes of linear fits over 7 or 8 time data, while only 4 points were available before.

This improves our confidence in the frequency estimation.

The closing date (MJD = 50689, 29 August 1997) corresponds to the last date included in the last complete two-month interval of computation available when writing this paper with the intention to implement the new algorithm in January 1998.

For this exercise the upper limit of relative weight was set at 0.8%, which corresponds closely to the upper limit of relative weight given to the best clocks in November-December 1995.

The time scale E is compared with the published EAL over the period under study. A time scale TE is deduced from E using the steering frequency corrections applied to EAL to obtain TAI. TE is then comparable with the published TAI over the same period.

Several other time scales, using only one of the two changes described here or testing other possibilities, have been computed in parallel with E. The time scale E is the one which led to the most significant results, this being the reason why it is recommended here.

RESULTS

STABILITY

Values of the stability of the time scales E and EAL are estimated by application of the 3-cornered-hat technique to data obtained from January 1996 to August 1997 in comparisons between E (or EAL) and two of the best time scales in the world, maintained at the NIST and at the USNO. This leads to the values for the Allan standard deviation $\sigma_y(\tau)$ shown in Fig. 3.

Fig. 3 calls for some remarks:

- Stability is estimated for averaging times not exceeding 80 d. Evaluating Allan standard deviation values, with reasonable confidence, for longer averaging times is very difficult: we have at our disposal only 20 months of data and the time scales considered may be subject to variations which are correlated in the long term, a circumstance sufficient to prevent the use of the *N*-cornered-hat technique. It is probable, however, that E and EAL are subject to a residual annual variation which would appear in Fig. 3 as a 'bump' for averaging times of about 180 d.
- The time scales E and EAL present white frequency noise for averaging times between 5 and 40 days, with no residual trace of noise coming from time transfer methods.
- The time scale E is more stable than EAL for all averaging times*.
- The best performance is:

$$\sigma_{y\mathrm{EAL}}(\tau = 40\ \mathrm{d}) = 1.3 \times 10^{-15},$$
$$\sigma_{y\mathrm{E}}(\tau = 40\ \mathrm{d}) = 1.1 \times 10^{-15}.$$

* *Another stability estimation computed with the 9-cornered-hat technique, involving the 8 best HP 5071A clocks reported to the BIPM, gives the same result.*

Accuracy

To characterize the accuracy of TE and TAI, estimates are made of the relative departures d_T, and of their uncertainties σ_T, of the durations of the TE and TAI scale intervals, u_T, from the SI second, u_0, as produced on the rotating geoid by primary frequency standards:

$$d_T = \frac{u_T - u_0}{u_0},$$

with T = TE or TAI.

Since January 1996, individual measurements of the TE and TAI frequencies have been provided by five primary frequency standards:

- LPTF-FO1, which is a cesium fountain developed at the BNM-LPTF, Paris, France. The preliminary evaluation of its accuracy led to a type B standard uncertainty of 3×10^{-15}, a value never reached before. Three measurements taken in May 1996 and averaged over periods of about 10 hours were sent to the BIPM.
- NIST-7, which is the optically pumped primary frequency standard developed at the NIST, Boulder, Colorado, USA. In the period covered by this report, it provided four measurements which cover a 5 day period in March 1996 and three 10 day periods in May 1996, December 1996 and June 1997. The type B standard uncertainty of NIST-7 is 1×10^{-14} for the first two measurements and 7×10^{-15} for the last two measurements.
- PTB CS2 and PTB CS3, which are classical primary frequency standards operating continuously as clocks at the PTB, Braunschweig, Germany. Frequency measurements are taken continuously and can be reported over successive one-month or two-month periods. The type B standard uncertainties
- SU MCsR 102, which is a classical primary frequency standard operated at the VNIIFTRI, Moscow, Russia. It delivered two measurements, both averaged over two-month periods, in February and March 1996. The type B standard uncertainty of this standard is 5×10^{-14}.

Values of d_{TE} and d_{TAI} deduced from these individual measurements are reported in Figs. 4 and 5, where results from PTB CS2 and PTB CS3 are treated over one-month intervals to assess the accuracy of TE, and over two-month intervals for TAI. The number of points in Fig. 5 is thus larger than in Fig. 4. Values over one-month intervals are not available for SU MCsR 102, which explains that the number of points concerning this standard is the same in both figures. Points deduced from LPTF-FO1, NIST-7 and SU MCsR 102 cannot be exactly superposed in the two figures because they are not related to the same time scale.

The uncertainty of each point in Figs. 4 and 5, except those from LPTF-FO1, is close to the type B uncertainty of the primary frequency standard since the uncertainty caused by the transfer to TAI is negligible. For LPTF-FO1, an additional uncertainty of about 5×10^{-15} must be taken into account for the link to TAI.

Estimates of TE and TAI accuracy obtained by global treatment of individual measurements [8] are added in Figs. 4 and 5. Measurements from PTB CS3 are not used in the processing because this standard experienced frequency steps of several parts in 10^{14} over the period under study. This global treatment can provide mean values of d_T estimated over durations of one or two months. The continuous lines of Figs. 4 and 5 correspond to two-month estimates of d_{TAI} and d_{TE} and are thus directly comparable. They appear to be nearly identical, the largest discrepancy between the two is

observed over July-August 1997:

$$d_{TE} = 1.7 \times 10^{-14},\ \sigma_{TE} = 1.0 \times 10^{-14},$$
$$d_{TAI} = 1.8 \times 10^{-14},\ \sigma_{TAI} = 1.0 \times 10^{-14},$$

and is much smaller than the corresponding uncertainties. Over the whole period under study the obtained values of σ_{TE} and σ_{TAI} are very close to each other and vary from 0.6×10^{-14} to 1.0×10^{-14}.

The accuracy of the time scale is thus nearly unchanged by the alternative algorithm. The discrepancy obtained is close to that resulting from uniform application of the correction for the black-body radiation frequency shift in 1995, for which a procedure for compensation was applied immediately (cumulative frequency steering corrections, each of relative amplitude 1×10^{-15} applied on dates separated by 60-day intervals). Current results suggest that this procedure has compensated only for the natural drift of the scale and that it should be reinforced, keeping in mind that the middle-term stability of the scale should not be degraded. For this reason successive frequency steering corrections of greater amplitude, 2×10^{-15}, have been applied since May 1997. This conclusion does not depend on the choice of algorithm.

CONCLUSIONS

We propose to the CCTF Working Group on TAI the implementation of an alternative algorithm for TAI computation. This algorithm is based on the same defining equations as the one in use at present, but includes two changes: shortening of the computation time of the time scale from two months to one month and use of an upper limit to the relative weights attributed to contributing clocks. Tests show that the middle-term stability of the resulting scale is improved and that its accuracy is unchanged, when compared with the published TAI. In addition, the delay of access to the time scale is reduced and the algorithm is more robust in response to global changes in the quality of the clocks, of the kind we experienced since the entry of the HP 5071A units into the computation.

The BIPM is ready to implement this alternative algorithm for the computation covering the month of January 1998.

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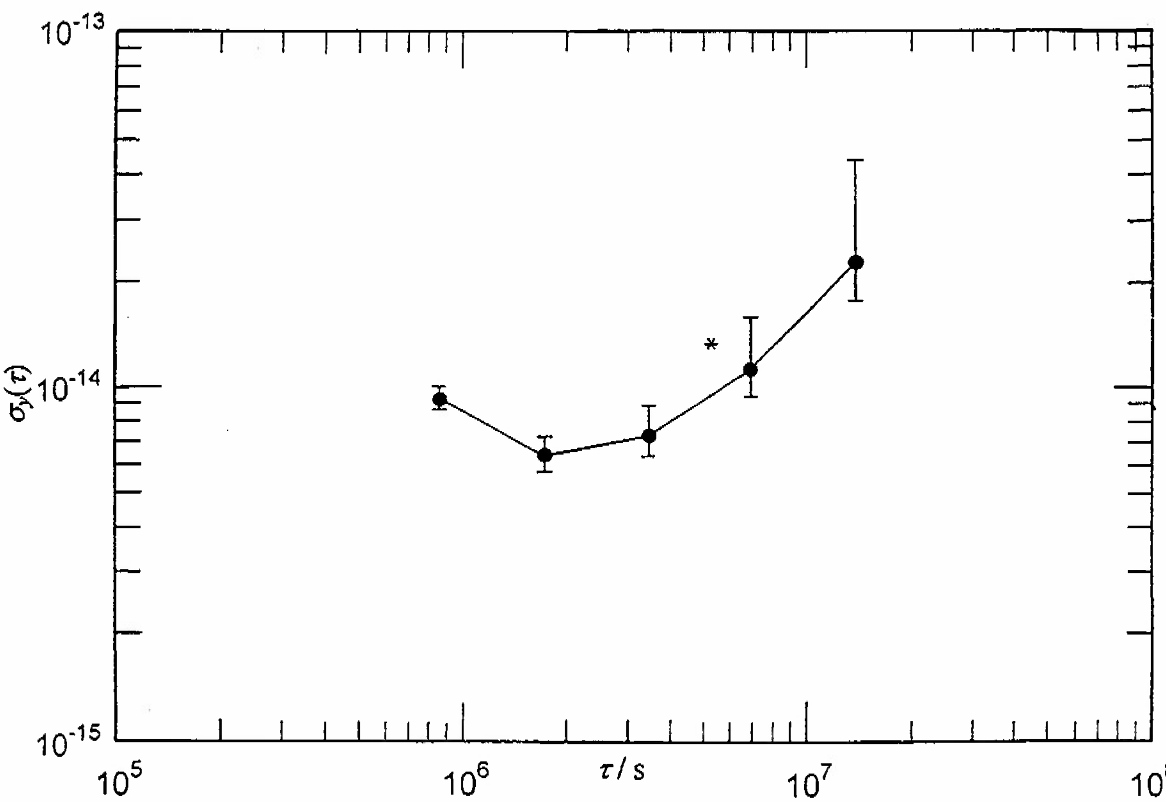

Figure 1. Typical stability curve of a HP 5071A clock in terms of variation of the Allan standard deviation σ_y with the averaging time τ. $*$ indicates the value $\sigma_y(\tau = 60\ \mathrm{d})$ sufficient for a given clock to be assigned the upper limit of absolute weight p_{MAX} in TAI computation.

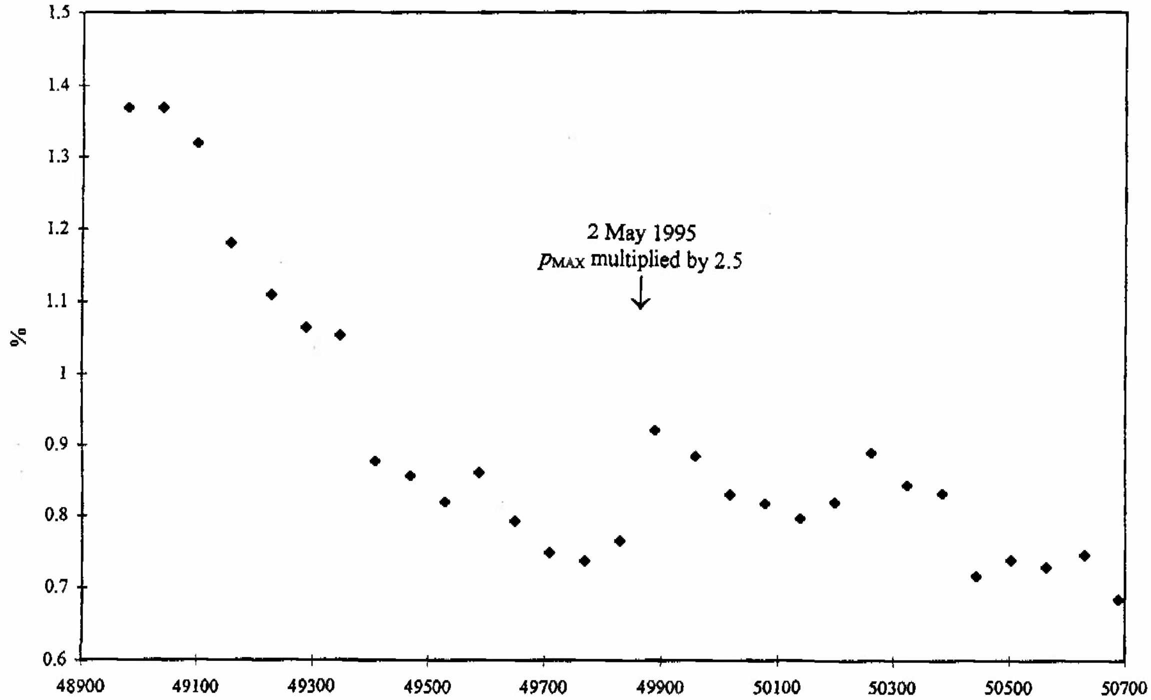

Figure 2. Maximum relative contribution ω_{MAX} taken by an individual clock in TAI computation from January 1993 to

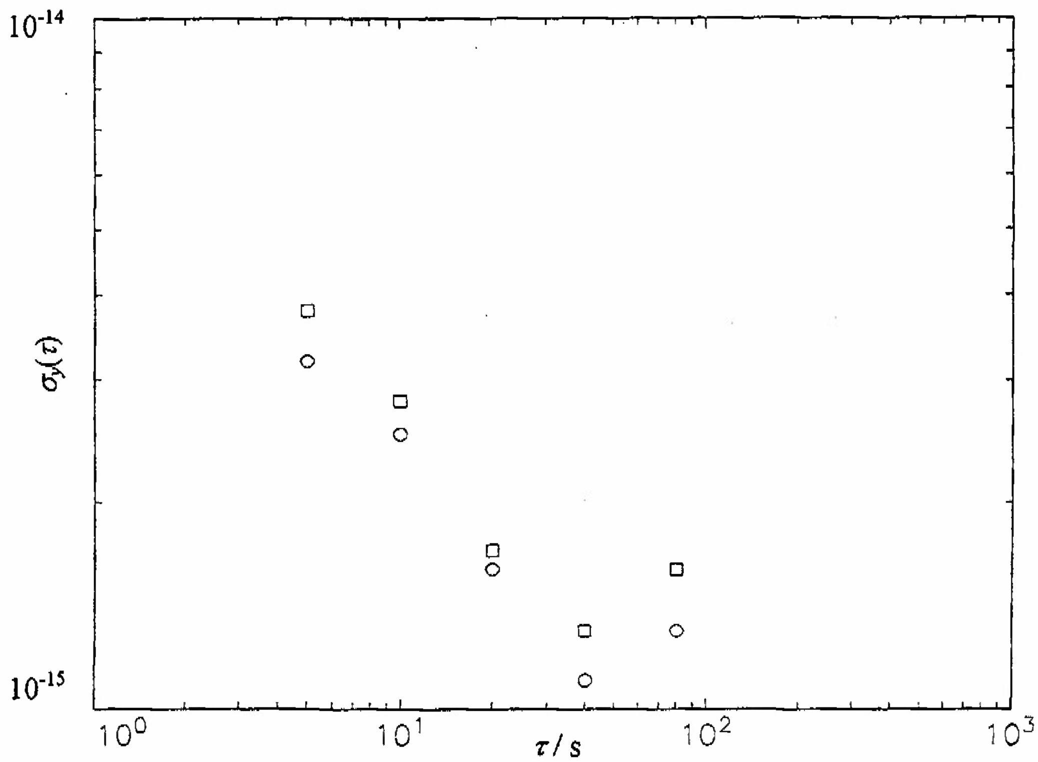

Figure 3. Stability of the time scales E (O) and EAL (□) estimated by application of the 3-cornered-hat technique to data obtained from January 1996 to August 1997 in comparisons between the time scales E or EAL, AT1 (maintained at the NIST), and A.1MEAN (maintained at the USNO).

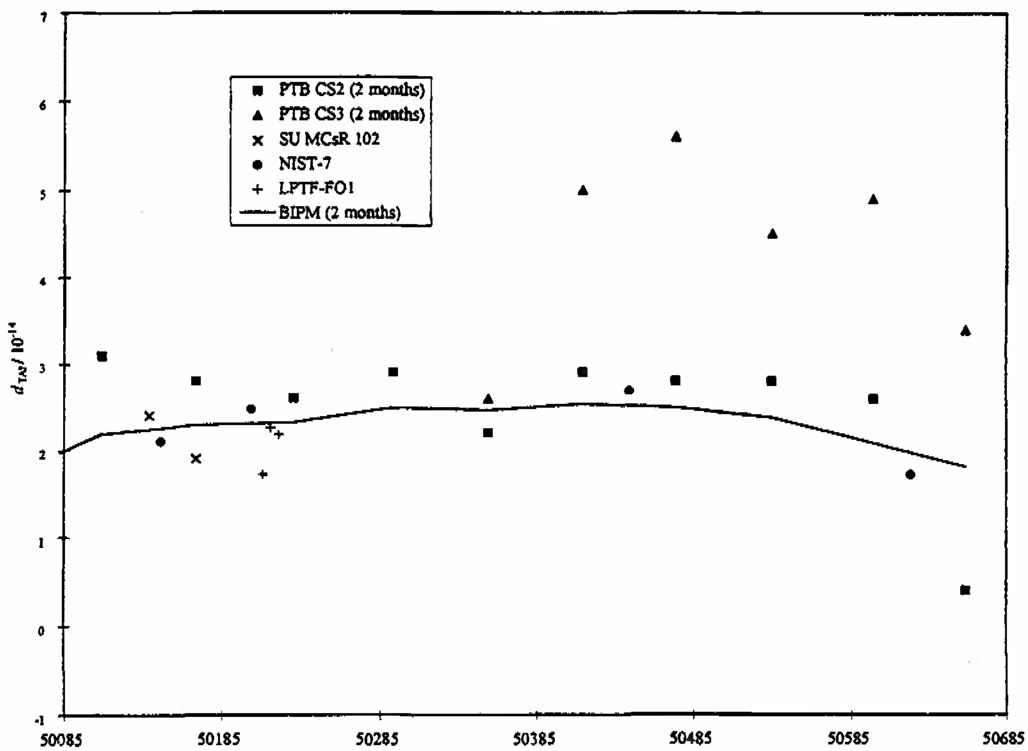

Figure 4. Accuracy of TAI over the period January 1996 to August 1997.
The quantity reported is the relative departure, d_{TAI}, of the duration of the TAI scale interval from the SI second as produced on the rotating geoid by primary frequency standards. The continuous curve corresponds to BIPM estimates of d_{TAI} over two-month intervals.

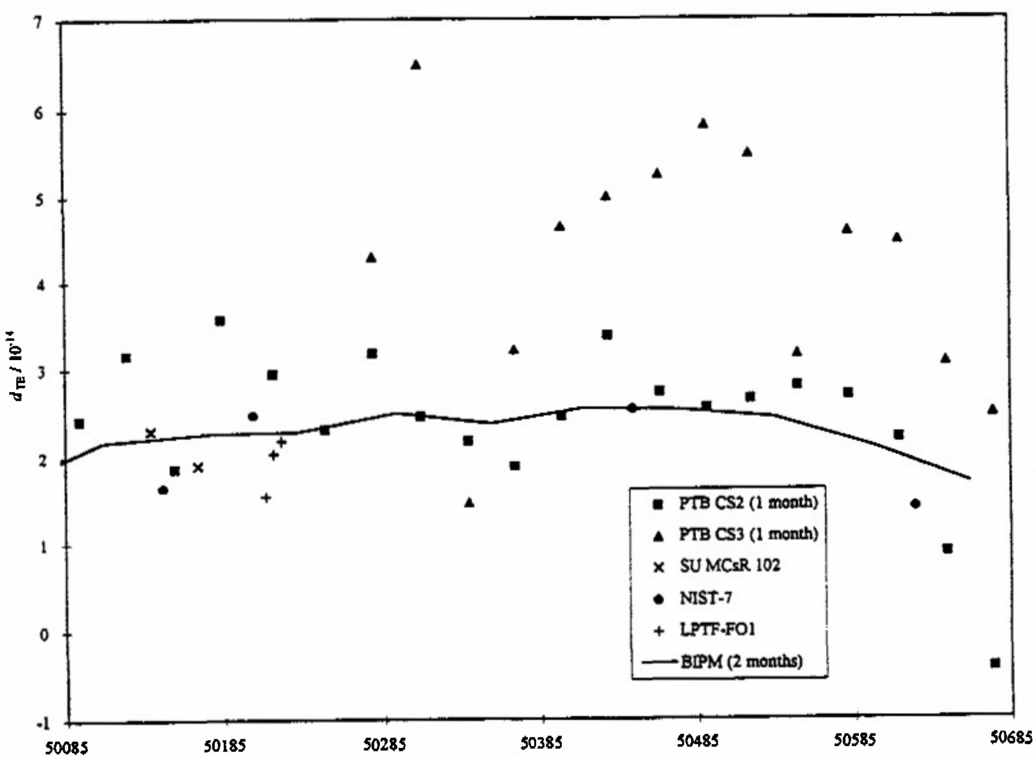

Figure 5. Accuracy of TE over the period January 1996 to August 1997.
The quantity reported is the relative departure, d_{TE}, of the duration of the TE scale interval from the SI second as produced on the rotating geoid by primary frequency standards. The continuous curve corresponds to BIPM estimates of d_{TE} over two-month intervals.

Questions and Answers

JUDAH LEVINE (NIST): We think it is a very good idea. We have said that to BIPM in writing. I think we would not be too concerned about the possible correlation between AT-1 and EAL because we have a relatively small number of clocks that contribute to EAL, and I would guess that the correlation between AT-1 and EAL is very small. I think the results of the three-corner hat using AT-1 are just fine.